* First order First degree D.E.

* Derivative Form.

$$a_1(x)\,y' + a_0(x)\,y = g(x)$$

$$\Rightarrow \boxed{\frac{dy}{dx} + P(x)\,y = Q(x)}$$

General form $\Rightarrow$ $f^n(y', y, x)$

* $\boxed{\text{Method of Solution}}$

① Seperation of Variable:-

$$X\,dx + Y\,dy = 0$$

* Solve The D.E. ....

ex- $\dfrac{dy}{dx} = \dfrac{y}{x^2}$

$\dfrac{dy}{y} = \dfrac{dx}{x^2}$ $\therefore \displaystyle\int \frac{1}{y}\,dy = \int \frac{1}{x^2}\,dx$

$\ln y = \dfrac{-1}{x} + C$ $\Rightarrow$ $C = \ln A$

$\ln y = \dfrac{-1}{x} + \ln A$ $\therefore$ $\ln y - \ln A = \dfrac{-1}{x}$

$\therefore \quad y = A e^{-\frac{1}{x}}$

$y = A e^{-\frac{1}{x}} \rightarrow (1)$

لما تلاقي $\ln y$ طرف ... تكتبه الصوره دي

ex2 : $y' = \frac{x+1}{y^4+1}$

$\frac{dy}{dx} = y'$

$\therefore \int (y^4+1)\,dy = \int (x+1)\,dx$

$\therefore \frac{y^5}{5} + y = \frac{x^2}{2} + x + C$

ex3 :- $(x^2+4)\frac{dy}{dx} = xy$

$\therefore \int \frac{1}{y}\,dy = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx$

$\ln y = \frac{1}{2}\ln(x^2+4) + C_{\ \to = \ln A}$

$\therefore \ln y = \ln (x^2+4)^{1/2} + \ln A$

$\ln\left(A(x^2+4)^{1/2}\right)$ نعطي e للطرفين

$y = A\sqrt{x^2+4}$

Note

$\int x e^{x^2}\,dx = \text{---}$

	تفاضل	تكامل
$+$	x	e^{x^2}
$-$	1	$\frac{1}{2x} e^{x^2}$
$+$	0	$\frac{e^{x^2}}{4x^2}$

ex4: $xy\,dx + e^{-x^2}(y^2-1)\,dy = 0 \qquad \div (y \cdot e^{-x^2})$

$\int e^{x^2} * x\,dx + \int \left(\frac{y^2-1}{y}\right) dy = 0$

$\frac{1}{2}\int 2 * x\, e^{x^2}\,dx + \int \left(y - \frac{1}{y}\right) dy = 0$

$\frac{1}{2} e^{x^2} + y^2/2 - \ln y = C$

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ex 5 : $\frac{dy}{dx} + p(x)\, y = 0$

$\int \frac{1}{y}\, dy = -p(x)\, dx$ نضرب الـ $\frac{1}{y}$

$\therefore y = C e^{-p(x)}$ حسب ①

2 : Can be Seperable

$y' = f^n(ax + by + c)$

Put $u = ax + by + c$ $\therefore u' = a + b y'$

$\therefore y' = \frac{u' - a}{b}$

$\because y' = f^n(ax + by + c)$

$\therefore \frac{u' - a}{b} = f^n(u)$

$\therefore u' = b f^n(u) + a$

$\therefore \frac{du}{dx} = b f^n(u) + a$ $\therefore \int \frac{1}{b f^n(u) + a}\, du = \int dx$

$u = ax + by + c$ استبدل

Q: 1 $\quad y' = (x+y+1)^2$

Solution

$\Rightarrow$ Can be Sep.

put $u = x+y+1$

$\therefore y' = u^2 \qquad \Rightarrow \dfrac{1}{b\,f(u)+a}\,du = dx$

بالتعويض

$\therefore \displaystyle\int \frac{1}{u^2+1}\,du = \int dx$

$\therefore \tan^{-1} u = x + C$

أو نحسب الـ u' الأول بدل ما نعوض في القانون...

$u = x+y+1 \qquad u' = 1 + y'$

$\therefore y' = u - 1$

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EX 2 : $y' = \tan^2(\underbrace{x+y+6}_{=u})$

$\therefore y' = \tan^2(u)$

بالتعويض المباشر

$\therefore \int \frac{du}{\tan^2 u + 1} = \int dx$

$\therefore \int \frac{du}{\sec^2 u} = \int dx \qquad \therefore \int \cos^2 u \, du = \int dx$

$\cos^2 u = \frac{1}{2} + \frac{1}{2}\cos(2u)$

$\therefore \int \left(\frac{1}{2} + \frac{1}{2}\cos(2u)\right) du = dx$

$\therefore \frac{1}{2}u + \frac{1}{4}\sin 2u = x + c$

put $u = x + y + 6$

3 : Linear method

$$y' = P(x)y = Q(x)$$

Sol

$Q(x) = 0$ نفرض مساوية

$\therefore y' + P(x)y = 0$

$\therefore \frac{dy}{dx} = -P(x)\,y \qquad \therefore \int \frac{1}{y}\,dy = \int -P(x)\,dx$

$$\therefore y = e^{-\int P(x)\,dx} \cdot C$$

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$$\therefore \quad y\, e^{\int P(x)\,dx} = C$$

$$\frac{d}{dx}\left(y \cdot e^{\int P(x)\,dx}\right) = 0 \longrightarrow (1)$$

تفاضل

$$y \cdot P(x)\, e^{\int P(x)\,dx} + y'\, e^{\int P(x)\,dx} = 0$$

$$\therefore \quad \left(y' + P(x)\, y\right) e^{\int P(x)\,dx} = 0 \longrightarrow (2)$$

$y' + P(x)\,y$ مكافئه ← من الصوره العامة

$$\Rightarrow \left[y' + \underbrace{P(x)\, y}\right] e^{\int P(x)\,dx} = Q\, e^{\int P(x)\,dx}$$

تكامل الطرفين

$$\Rightarrow \int \frac{d}{dx}\left(y \cdot e^{\int P(x)\,dx}\right) = \int Q \cdot e^{\int P(x)\,dx}$$

$$\Rightarrow \quad y\, e^{\int P(x)\,dx} = \left[\int Q\, e^{\int P(x)\,dx} + C\right]$$

$$y = \frac{1}{e^{\int P(x)\,dx}} \left[\int Q\, e^{\int P(x)\,dx} + C\right]$$

Put $e^{\int P(x)\,dx} = I$

$$\therefore \quad y = \frac{1}{I}\left(\int Q\, I\, dx + C\right)$$

Ex:1 $y' \cancel{+} (\frac{-3}{x}) y = 1$

Sol

$Q = 1$

$P(x) = \frac{-3}{x}$

$\Rightarrow I = e^{\int (\frac{-3}{x}) dx} = e^{-3\ln x} = e^{\ln(x)^{-3}} = \frac{1}{x^3}$

$y = \frac{1}{I}\left[\int (1)(\frac{1}{x^3})\, dx + c\right]$

$y = x^3\left[\frac{-x^{-2}}{2} + c\right] = \frac{-x}{2} + cx^3$

Ex:2 $\tan x \frac{dy}{dx} + y = \sin^2 x$ $\div \frac{\cos x}{\sin x}$

$\therefore y' + \frac{\cos x}{\sin x} y = \sin x \cos x$

$\Rightarrow P(x) = \frac{\cos x}{\sin x}$

$\Rightarrow Q(x) = \sin x \cos x$

$\therefore I = e^{\int (\frac{\cos x}{\sin x}) dx} = e^{\ln(\sin x)} = \sin x$

$y = \frac{1}{I}\left[\int (\sin x \cos x)(\sin x)\, dx + c\right]$

$\left[\int (\sin^2 x \cos x\, dx) + c\right]$

$y = \frac{1}{\sin x}\left[\frac{\sin x^3}{3} + c\right]$

ex1 $y' + \tan y = \sin x$

Solution

$P(x) = \frac{\sin x}{\cos x}$ $\qquad Q(x) = \sin x$

$I = e^{\int p(x)dx}$

$I = e^{-\int \frac{-\sin x}{\cos x} dx} = e^{\ln(\cos x)^{-1}} = \frac{1}{\cos x}$

$y = \frac{1}{\frac{1}{\cos x}} \left[-\int \frac{-\sin x}{\cos x} dx + c \right]$

$y = \cos x \left[\ln(\cos x)^{-1} + c \right]$ #

ex:2 $x \ln x \ln\ln x \; y' + y = (1+\frac{2}{x}) \ln x$

Final

Solution

$y' + \frac{1}{x\ln x \ln\ln x} y = \frac{(1+\frac{2}{x})}{x \ln\ln x}$

$\Rightarrow P(x) = \frac{\frac{1}{x\ln x}}{\ln\ln x}$ $\qquad Q(x) = \frac{1+\frac{2}{x}}{x\ln\ln x}$

$I = e^{\int \frac{1/x\ln x}{\ln\ln x}} = e^{\ln\ln\ln x} = \underline{\ln\ln x}$

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$$\Rightarrow y = \frac{1}{\ln\ln x}\left[\int \frac{1+2\ln x}{x\ln\ln x}\,\ln\ln x\,dx + c\right]$$

$$y = \frac{1}{\ln\ln x}\left[\int\left(\frac{1}{x} + \frac{2}{x^2}\right)dx + c\right]$$

$$= \frac{1}{\ln\ln x}\left[\ln x + \frac{-2}{x} + c\right]$$

$\boxed{4}$:- Can be linear :- "Bernoulli eq"

$$y' + P(x)\,y = Q(x)\,y^n$$

$y^n \div$ [illegible] (Sol)

$$\therefore \Rightarrow \frac{1}{y^n}\,y' + P(x)\,y^{1-n} = Q(x) \rightarrow *$$

$$\Rightarrow \text{put } \boxed{z = y^{1-n}} \rightarrow ①$$

$$\therefore z' = (1-n)\,y^{-n}\,y'$$

$$\therefore \boxed{z' = (1-n)\,y^{-n}\,y'} \Rightarrow \frac{z'}{1-n} = \frac{y'}{y^n} \rightarrow ②$$

* [illegible]

$$\Rightarrow \therefore \quad \therefore \frac{z'}{1-n} + P(x)\,z = Q(x)$$

$$\therefore z' + (1-n)\,P(x)\,z = (1-n)\,Q(x)$$

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$$z' + (1-n)\, P(x)\, z = (1-n)\, Q(x)$$

$$\Rightarrow \therefore I = e^{\int (1-n) P(x) dx}$$

$$\therefore y^{1-n} = z = \frac{1}{I}\left[\int I\,(1-n)\, Q(x)\, dx + c\right]$$

ex 1: $(1+x^2)y' - 2xy = x e^{-x} y^3$

Solution

$$\Rightarrow y' + \frac{-2x}{(1+x^2)}\, y = \frac{x e^{-x}}{(1+x^2)}\, y^3$$

$$P(x) = \frac{-2x}{1+x^2} \qquad Q(x) = \frac{x e^{-x}}{1+x^2}$$

$$\Rightarrow I = e^{\int \frac{-2x}{1+x^2}(1-3)\, dx} = e^{2\int \frac{2x}{1+x^2} dx}$$

$$\therefore I = e^{\ln(1+x^2)^2} = (1+x^2)^2$$

$$\Rightarrow z = \frac{1}{(1+x^2)^2}\left[\int -2(1+x^2)^2 \times \frac{x e^{-x}}{(1+x^2)}\, dx + c\right]$$

$$z = \frac{-2}{(1+x^2)^2}\left[\int (1+x^2)\, x e^{-x}\, dx + c\right]$$

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$z = \frac{-2}{(1+x^2)^2} \left[\int (x e^{-x} + x^3 e^{-x}) dx + c \right]$

$z = \quad \left\{ -e^{-x} + c \right.$

x^3	e^{-x}
$-3x^2$	$-e^{-x}$
$+6x$	$+e^{-x}$
-6	$-e^{-x}$
-0	$+e^{-x}$

5 : Homogenous Method

From $M(x,y)dx + N(x,y)dy = 0$

$\Rightarrow$ Put $y = vx$ $\qquad v = \frac{y}{x}$

$y' = v + x\frac{dv}{dx}$

$x=1$ $x y^2 y' = x^3 + y^3$

$f(x,y) = x y^2$

$f(\lambda x, \lambda y) = \lambda x (\lambda y)^2$

$\quad = \lambda x \; \lambda^2 y^2 = \lambda^3 x y^2$

f^n is Homo.

if $x^2 + y^2$

$x \lambda$

$= (\lambda x)^2 + (\lambda y)^2$

$= \lambda^2 x^2 + \lambda^2 y^2$

$= \lambda^2 (x^2 + y^2)$

$\therefore f^n$ is Homo.

$y' = \frac{x^3 + y^3}{x y^2}$

بالتعويض

$\therefore \quad v + x\frac{dv}{dx} = \frac{x^3 + v^3 x^3}{x\, v^2 x^2}$

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$$v + x\frac{dv}{dx} = \frac{1+v^3}{v^2}$$

$$\cancel{v} + x\frac{dv}{dx} = \frac{1}{v^2} + \cancel{v}$$

$$\therefore \int v^2\,dv = \int \frac{1}{x}\,dx \qquad \therefore \frac{v^3}{3} = \ln x + C$$

$$v = \frac{y}{x}$$

ex2 $y' = \frac{x^2+y^2}{xy}$

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{x\,vx} = \frac{1+v^2}{v}$$

$$\therefore \cancel{v} + x\frac{dv}{dx} = \frac{1}{v} + \cancel{v}$$

$$\therefore \int v\,dv = \int \frac{1}{x}\,dx$$

$$\frac{v^2}{2} = \ln x + C$$